

MATH210 Spring 2015
Test 2

1. Prove the following for any natural number n :

$$\sum_{i=1}^n (5i - 1) = \frac{n(5n + 3)}{2}$$

2. (a) True or False: the sum of the first five terms of the arithmetic sequence with $a = 2$ and $d = 3$ is 39.

- (b) Let $a_0 = 7$ and $a_{i+1} = 2a_i - 3$. True or False: $a_4 = 63$.

3. Let $A = \{2k \mid k \in \mathbb{Z}\}$ and $B = \{n \mid n \in \mathbb{Z}, 4 \mid n^2\}$. Prove that $A = B$.

4. For sets A and B answer the following as true or false:

(a) $(A^c \cup B)^c = A \cap B^c$?

(b) $A \setminus B \subseteq A \oplus B$?

5. Let $S = \{1, 2, 3, 4\}$. List the ordered pairs in a binary relation on S which is reflexive, transitive, symmetric, contains $(1, 4)$, contains $(4, 2)$, and does not contain $(3, 2)$.

6. The set $S = \{1, 2, 3, 4\}$ can be partitioned into $T_1 = \{1, 2\}$ and $T_2 = \{3, 4\}$. Give the ordered pairs of the equivalence relation whose equivalence classes are T_1 and T_2 .

7. (a) Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be given by $f(x) = x + 2$. Is f onto?

(b) Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be given by $f(x) = x^2 + 1$. Is f one-to-one?

Extra Credit Find the first 7 perfect squares in the sequence $4, 15, 26, 37, \dots$ (Hint: 4 is the first and you don't have time to calculate all of them.)