

**MATH210 Spring 2015**  
**Test 1**

1. (a) Construct a truth table for the compound statement

$$(p \rightarrow q) \wedge (r \rightarrow q)$$

- (b) What is the truth value of  $(p \rightarrow q) \wedge (r \rightarrow q)$  when  $p$  is false,  $r$  is false, and  $q$  is true?

2. What is the contrapositive of  $(p \vee q) \rightarrow (r \wedge p)$ ?

3. Write the negation of the following statements:

(a) If  $x$  is an even integer then  $x^2$  is an even integer.

(b) Every planar graph can be colored with at most four colors.

(c)  $x > -2$  and  $x < 1$ .

4. Prove that  $n^3$  is odd if and only if  $n$  is odd.

5. Let  $a = -4607$  and  $b = 47$ . Find integers  $q$  and  $r$  with  $0 \leq r < |b|$ , such that  $a = bq + r$ .

6. (a) Express 27 as a binary number.

(b) Express 27 as an octal number.

7. Prove that the integers  $a$  and  $b$  can have at most one greatest common divisor.

8. Prove that  $\gcd(3k + 2, 5k + 3) = 1$  for any  $k \in \mathbb{N}$ .

**Extra Credit** Given any natural number  $n > 1$ , show that there is a prime number  $p$  that divides  $n$ .